Aeroelastic Oscillations Caused by Transitional Boundary Layers and Their Attenuation

D. G. Mabey,* P. R. Ashill,† and B. L. Welsh‡
Royal Aircraft Establishment, Bedford, England

Recent wind-tunnel tests on model wings show large-amplitude aeroelastic oscillations in the first bending mode associated with transitional boundary layers. A possible mechanism is suggested to explain the oscillations primarily in terms of reduced aerodynamic damping. The validity of the hypothesis is confirmed by further measurements in which the oscillations were attenuated by the installation of a vibration absorber in the hollow wing-tip body. Aeroelastic oscillations caused by transition have implications for the measurement of forces, buffeting and flutter characteristics in wind tunnels, and the operation of laminar-flow aircraft.

Nomenclature

A	= generalized mass in first bending mode
$c,\bar{c}$	= local and aerodynamic mean chords
C_L	= lift coefficient
f	= frequency, Hz
$g/2$	= structural damping coefficient, % critical
k	= constant, Eq. (A3)
m	= mass flow injected
m_b	= calculated mass flow in boundary layer just upstream of air holes
M	= Mach number
$\sqrt{nG(n)}$	= forcing function in first bending mode
q	= kinetic pressure
R	= unit Reynolds number
S	= wing area
U	= freestream velocity
x	= streamwise distance from leading edge
$y,\dot{y},\ddot{y}$	= wing-tip deflection, velocity, and acceleration
γ	= aerodynamic damping coefficient, % critical
ϵ	= rms wing root strain
ξ	= total damping coefficient, % critical
ν	= frequency parameter, $=2\pi f\bar{c}/U$
μ	= viscosity
ρ	= freestream density

Introduction

THE need to fix boundary-layer transition from laminar to turbulent flow on the surfaces of models used in wind tunnels is widely accepted and applies as well to routine and aeroelastic model tests.[1,2] Thus, the effects of scale between model and flight Reynolds number can be controlled and quantified. With "transitional" boundary layers, defined here as those in which transition occurs over a streamwise length that is a significant proportion of the local chord, large aeroelastic oscillations are sometimes observed. The origin of these aeroelastic oscillations is the subject of this paper.

During a recent wind-tunnel test[3] of a large aluminum half-model in which the boundary-layer transition was controlled carefully by blowing from small holes close to the leading edge[4] (at $x/c=0.05$ on both surfaces), violent oscillations occurred in the first wing bending mode when the boundary layer on the wing was not fixed. A preliminary analysis of these oscillations[5] suggested that this was an aeroelastic instability caused by a transitional boundary layer. For model safety, subsequent tests were made with the total damping in the wing first bending mode increased by a passive vibration absorber.[6] These new measurements suggest that in addition to reducing the aerodynamic damping, transitional boundary layers may magnify the aerodynamic excitation provided by flow unsteadiness in the wind tunnel, as suggested tentatively by measurements on a smaller swept-wing model[7].

The effects of a transitional boundary layer on the damping in the first bending mode would explain phenomena observed previously:

1) On a flutter model of a supercritical wing,[8] an anomalous flutter instability at the first bending frequency was observed at low equivalent airspeeds (EAS). It was attributed to "the transition strip becoming ineffective at low total pressure, leading to a variable transition location and a deviation in unsteady airloads."

2) In a transonic test,[9] a low-aspect-ratio flutter model was destroyed by an aeroelastic oscillation at low EAS.

3) Anomalies were observed at transonic speeds when making tests with free transition on a flutter model of the highly swept fin of the Concorde aircraft.[10] These phenomena were eliminated when the tests were made with fixed transition.

4) Boundary-layer transition is known to affect the dynamic stability in pitch of full-scale re-entry bodies at high supersonic speeds.[11] When the boundary layer is either purely laminar or turbulent, the effects of changes in the boundary-layer thickness are small. However, when transition occurs, the boundary-layer thickness effects become appreciable and cause large reductions in the pitch damping.

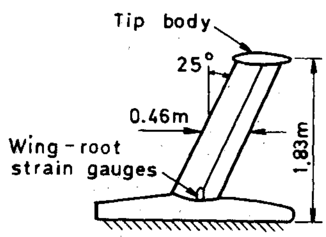
Experimental Details

An aluminum half-model (Fig. 1a) was tested in the RAE 8×8 ft tunnel to investigate the steady aerodynamic characteristics of wings with rear pressure rises of different form and severity.[3] This model could be provided with alternative wing profiles aft of 55% chord; otherwise, it had no joints. Hence, it was considered likely to have low structural damping and significant aerodynamic damping.[12] The wing first bending frequency was 10 Hz. Figure 1a shows the position of the wing-root strain gages installed to measure the bending vibrations as rms strain fluctuations.

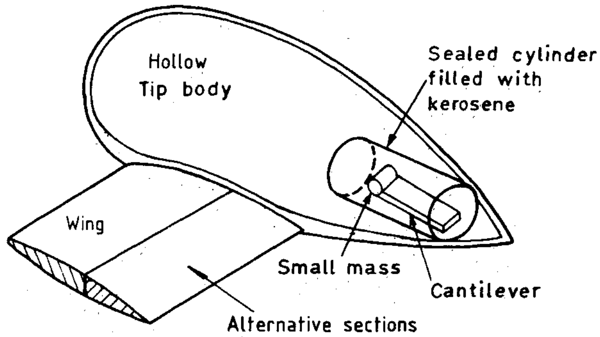
As shown in Fig. 1a, the model incorporated a large, hollow wing-tip body designed to maintain the sweep of the isobars in the tip region. This hollow body was utilized when a passive vibration absorber was developed for the later tests. A small steel cantilever was installed in the tip body

Received Dec. 24, 1986; revision received Jan. 5, 1987.

*Senior Principal Scientific Officer, Aerodynamics Department.
†Principal Scientific Officer, Aerodynamics Department.
‡Senior Scientific Officer, Aerodynamics Department.



a) Aluminum half-model.



b) Modified tip model.

Fig. 1 Details of model.

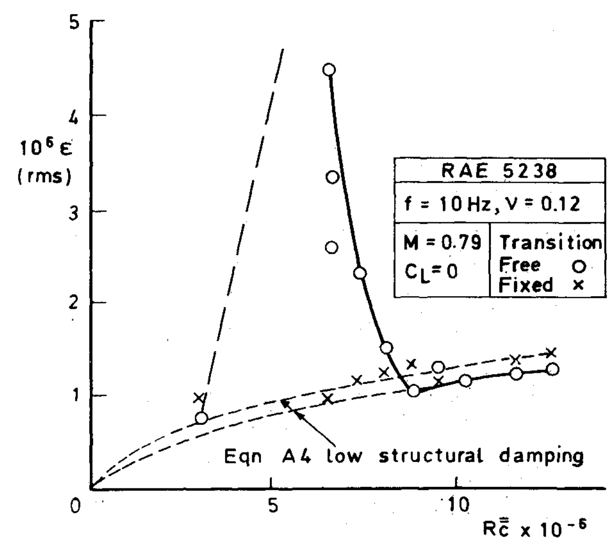


Fig. 2 Variation of unsteady wing-root strain with Reynolds number.

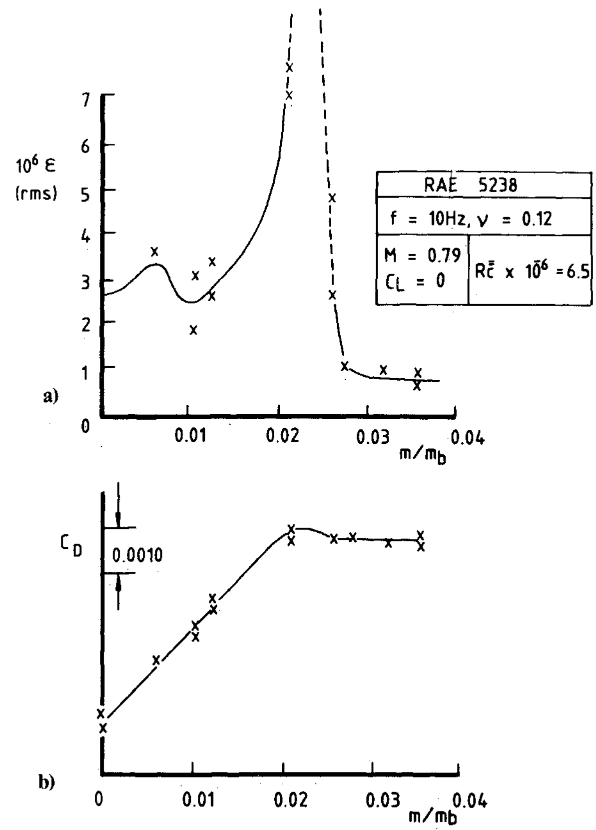


Fig. 3 Variation of unsteady wing-root strain ϵ and drag coefficient C_D with air injection ratio.

(Fig. 1b). One end of the cantilever was screwed into the rear of the tip body and a small mass on the other end was adjusted until the cantilever/mass bending frequency was the same as the wing bending frequency (10 Hz). Then, a small cylinder within the tip body was partially filled with kerosene (as in the patent specification[6]) so that the vibration of the cantilever dissipated energy and produced an increase in the damping of the overall system. (Den Hartog[13] has discussed the modeling of these systems.) The increase in damping depends directly upon the viscosity of the fluid, the section and widths of the cantilever, and the width of the gap between the cantilever and the tip body.

All of the sections were symmetric and tested at zero incidence. Emphasis was placed on relating the rear pressure rise to the boundary-layer drag measured by a wake traverse at 67% semispan. For this purpose, transition was normally fixed at 5% chord on both surfaces, with air injected into the (laminar) boundary layer through a large number of small holes, each of diameter 0.25 mm and drilled normal to the wing surface. The effect of air injection can be expressed best[4] in terms of the mass flow ratio m/m_b, where m is the mass flow injected and m_b the calculated mass flow in the boundary layer just upstream of the air holes. As m/m_b increases, there is a rapid increase in drag, consistent with transition moving upstream to the air holes (see Fig. 3 below). At a certain value of m/m_b (usually between about 0.02 and 0.03), further increases in m/m_b result in no further increase in drag, at least up to about $m/m_b = 0.05$. In view of this, and in order to ensure that the transition is fixed properly, "transition-fixed" tests are normally made for values of m/m_b greater by about 0.01 than the values at the "corner" of the drag vs m/m_b curve.

Wing Response Measurements

During steady experiments, some degree of excitation inevitably occurs due to unsteadiness of the tunnel flow.[14] Figure 2 shows the variation with Reynolds number of the rms fluctuations of wing-root strain measured at $M = 0.79$ on the section considered to have the most extreme tendency toward separation at the trailing edge (section RAE 5238). With transition fixed at 5% chord by the air-injection system, the level of fluctuations caused by tunnel flow unsteadiness follows the variation with Reynolds number according to Eq. (A4), supporting the hypothesis that structural damping was low. In marked contrast, with no air injection and therefore free transition, a marked oscillation develops as the Reynolds number increases. The oscillation reaches a maximum at about $R\bar{c} = 6 \times 10^6$ and then decreases rapidly. At still higher Reynolds numbers when the transition moves closer to the leading edge, the model amplitude increases again according to Eq. (A4). These response measurements correlate quite well with the drag measurements (not presented), insofar as the latter suggest that at $R\bar{c} = 6 \times 10^6$ natural transition is at 30–40% chord and moves forward rapidly with the increase in the Reynolds number.

Figure 3 shows the results from a different type of investigation, made at the same Mach number but at a constant Reynolds number of $R\bar{c} = 6.5 \times 10^6$. The variation in

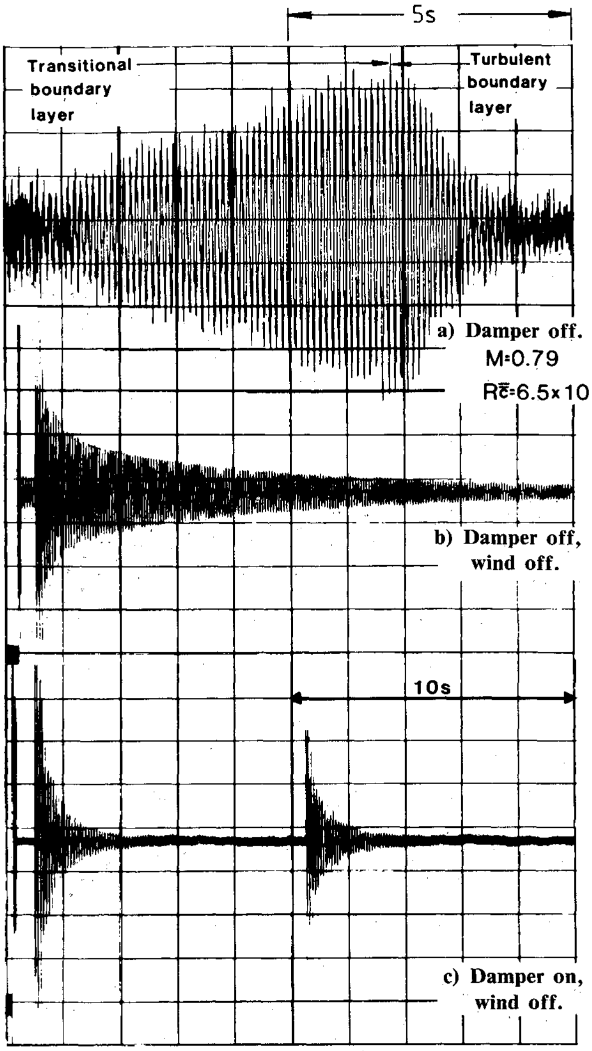


Fig. 4 Typical traces from wing-tip accelerometer.

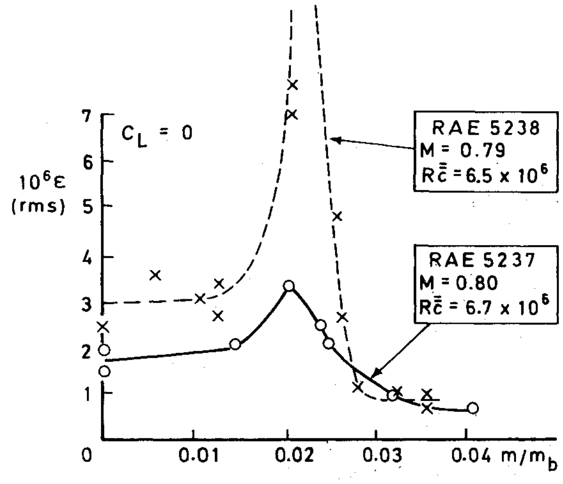


Fig. 5 Variation of unsteady wing-root strain with air-injection ratio for two sections.

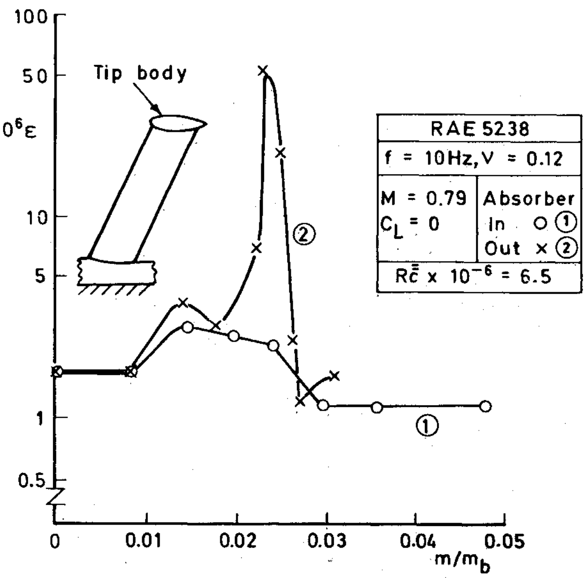


Fig. 6 Variation of unsteady wing-root strain with air injection ratio with vibration absorber in and out.

fluctuations is shown as the air injection mass flow is increased (Fig. 3a) and the variation in drag (Fig. 3b) suggests that transition moves upstream from an initial position at about $x/c = 0.30$ until fully turbulent conditions are obtained just downstream from the air-injection holes, as indicated by the constant level of drag obtained from the wake traverse.[3] The fluctuation measurements indicate again an increase in model amplitude as the mean transition position moves upstream from the natural position toward the air holes. With transition close to the air holes, the model amplitude tends to a limiting value roughly the same as at a mass flow ratio a little higher than that for constant drag. This suggests that the increased response is associated with an oscillatory chordwise movement of the boundary-layer transition front.

Some tape recordings of the fluctuations were taken during the second series of tests on the model; Fig. 4 illustrates the damping variations observed. Figure 4a shows the signal obtained without the vibration absorber during the recording of the highest peak shown in Fig. 6 for $M = 0.79$, $R\bar{c} = 6.5 \times 10^6$ and $C_L = 0$ (a repeated measurement corresponding to Fig. 3). For the first part of the trace, the blowing ratio is $m/m_b = 0.023$ and the boundary layer is transitional. Here, the wing bending motion is divergent. After about 6 s, it was decided that the wing-tip motion (with an amplitude of about 25–50 mm) was excessive. The mass flow ratio was increased quickly to $m/m_b = 0.030$, to give a fully turbulent boundary layer. The wing bending motion immediately became convergent, the damping ratio being about 1.5% critical. With the vibration absorber fitted, no response signals were obtained wind-on (for comparison with Fig. 4a) because of a local fault on the magnetic tape. Figure 4b shows the wind-off signal without the vibration absorber due to a sudden impulse. The wing is lightly damped (only about 0.5% critical), so that the aerodynamic damping with a turbulent boundary layer must be 1% critical based on the wind-on measurement of 1.5% critical total damping. Figure 4c shows the wind-off signal with the vibration absorber. The wing is heavily damped (about 3.5% critical) so that the aerodynamic damping with a turbulent boundary layer (about 1% critical) would be a relatively small proportion of the total damping.

Reference 5 includes additional evidence for the association of increased response with the forward movement of the mean transition front, including some measurements made at $M = 0.50$ with section RAE 5238, which gave the largest response observed. In general, the response of those sections having less severe adverse pressure gradients close to the trailing edge was found to be smaller than those shown in Figs. 2 and 3, but it was still significant. This may be seen from the measurements on section RAE 5237 at $M = 0.80$ shown in Fig. 5.

Figure 6 (with a logarithmic scale for wing response) compares measurements at $M = 0.79$ for RAE section 5238 with and without the wing-tip vibration absorber. With the wing-tip vibration absorber (curve 1), the structural damping is

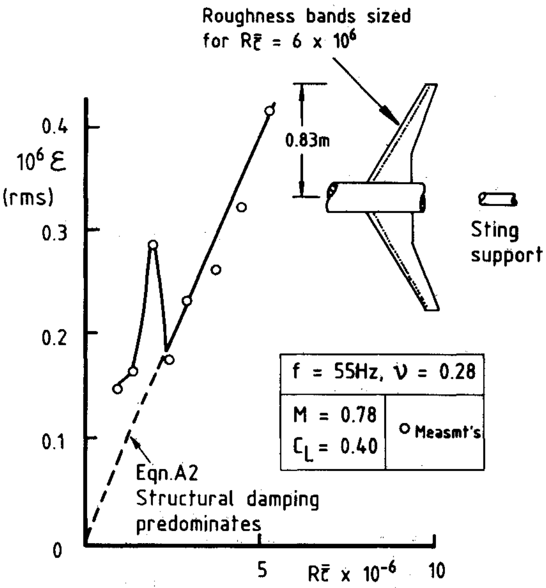
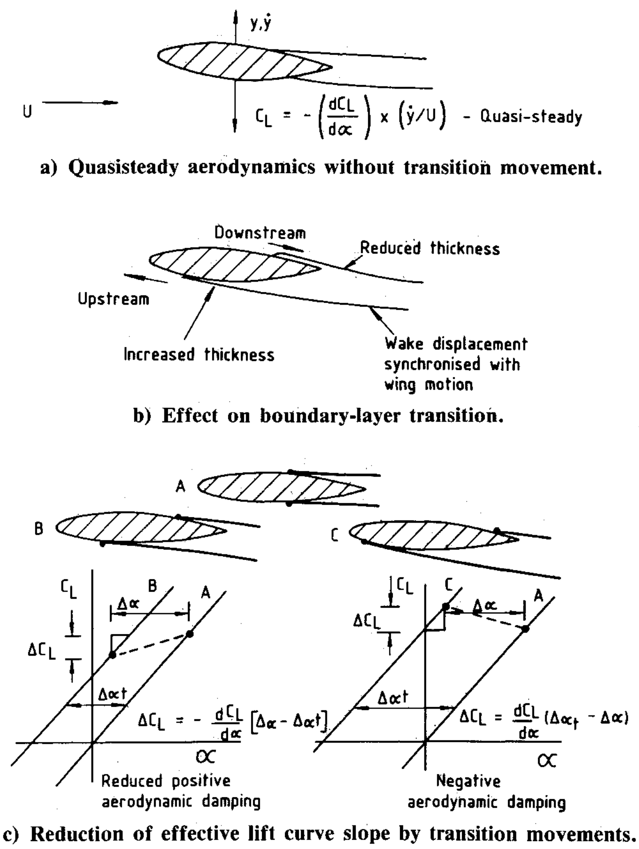


Fig. 7 Steel complete wing: variation of unsteady wing-root strain with Reynolds number.



a) Quasisteady aerodynamics without transition movement.

b) Effect on boundary-layer transition.

c) Reduction of effective lift curve slope by transition movements.

Fig. 8 Origin of damping variations.

greatly increased and hence the percent variation in total damping throughout the transition region is smaller. The doubling of the wing response in the transition region with the vibration absorber is due primarily to a large reduction in the total damping, due to negative aerodynamic damping. However, it is possible that the transitional boundary layer also magnifies the aerodynamic excitation provided by tunnel unsteadiness, as discussed with respect to Fig. 20 in Ref. 7. Without the wing-tip vibration absorber (curve 2, a repeated measurement to be compared with Fig. 3a), there is an increase in the response by an order of magnitude. This huge increase occurs because of an extremely low level of total damping in the limit cycle caused by the production of negative aerodynamic damping (see Fig. 4a).

Increases in fluctuations associated with transitional boundary layers are not restricted to aluminum half-models with low structural damping and high aerodynamic damping. Increases in response associated with transition were observed on a complete, sting-supported steel model in the RAE 8×8 ft tunnel.[5] With this model, structural damping was probably comparatively large relative to aerodynamic damping and, therefore, the increases in response caused by transitional boundary layers are less marked.

Figure 7 shows some measurements of fluctuations vs Reynolds number obtained with this model at $M = 0.78$ and $C_L = 0.4$. Roughness bands were placed at $x/c = 0.15$ on the upper surface and $x/c = 0.05$ on the lower surface. For Reynolds numbers higher than $R\bar{c} = 2.3 \times 10^6$, drag measurements (not presented) suggest that transition was effectively fixed at the roughness bands. Here, the response increases, for the most part, linearly with the Reynolds number according to Eq. (A2). This would imply that structural damping generally predominates, as was expected. Despite this, a small peak is observed at $R\bar{c} = 1.9 \times 10^6$. These measurements suggest that variations in aerodynamic damping caused by transition can occur on wings having totally different steady boundary-layer development on the upper and lower surfaces and with higher-frequency parameters.

Damping Variations Associated with Transitional Boundary Layers

These large aeroelastic oscillations are limit cycles caused by negative aerodynamic damping, which may be frequency and amplitude dependent. Within these tests, the frequency could not be varied and, hence, a full explanation is not yet possible. However, quasisteady considerations provide a simple explanation for the effect of transition on aerodynamic damping. Consider the section of a wing sketched in Fig. 8a, with transitional boundary layers on both surfaces. For a small upward velocity $\dot{y}$, the induced wing incidence is negative, $-\dot{y}/U$. Thus, the quasisteady lift coefficient is negative, providing a restoring force corresponding with positive aerodynamic damping. Compared with the stationary wing, the upward motion will induce stronger unfavorable pressure gradients on the lower surface and thus move the transition further upstream and give a thicker turbulent boundary layer at the trailing edge. Conversely, on the upper surface, the unfavorable pressure gradients will be less severe and, thus, transition will occur further downstream, giving a thinner boundary layer at the trailing edge (Fig. 8b). The boundary-layer thickness variations change the circulation around the wing by altering both the effective camber of the displacement surface[15] and, to a lesser extent, the wake curvature.[16] These would reduce the *magnitude* of the quasisteady lift and thus reduce the aerodynamic damping.

The quasisteady effects of the transition movements can be quantified in terms of effective lift curve (Fig. 8c). This suggestion is supported by limited comparative measurements of the overall lift on the wing with section RAE 5238 both with natural transition ($m = 0$) and transition complete. For zero lift, the lift curve slope with natural transition is close to zero, whereas with transition complete it is positive—as would be expected. Suppose that, with *fixed* transition on both surfaces at the *streamwise* position, the lift is given by curve A. Then, with *fixed* transition on both surfaces at *two different streamwise* positions, the lift-incidence curve will be displaced by increments $\Delta\alpha_t$ to curves B and C, due primarily to the negative camber of the boundary-layer displacement surface. During an upward motion of the section, with the transition varying from the fixed position of curve A, the lift must somehow be varied between curve A and curves B and C. For a quasisteady change in angle of incidence $\Delta\alpha$ (equivalent to $\dot{y}/U$ for the moving wing), the effective change in the angle of incidence becomes

$$\Delta\alpha - \Delta\alpha_t$$

Now to a first order in $\Delta\alpha_t$, the lift curve slopes of all the curves should be the same. Hence, the corresponding change in lift coefficient is only

$$\Delta C_L = \frac{dC_L}{d\alpha}(\Delta\alpha - \Delta\alpha_t) \tag{1}$$

and the effective lift curve slope, proportional to aerodynamic damping, is

$$\frac{\Delta C_L}{\Delta\alpha} = \frac{dC_L}{d\alpha}\left(1 - \frac{\Delta\alpha_t}{\Delta\alpha}\right) \tag{2}$$

Thus, the aerodynamic damping depends on the ratio $\Delta\alpha_t/\Delta\alpha$.

When $\Delta\alpha_t/\Delta\alpha \leq 1$ (curve B, Fig. 8c), the aerodynamic damping will remain positive, whereas if $\Delta\alpha_t/\Delta\alpha \geq 1$ (curve C, Fig. 8c), the aerodynamic damping will become negative and, if it overcomes the structural damping, single-degree-of-freedom flutter will occur. If the effect of the transitional boundary layer is to excite single-degree-of-freedom flutter,[2] even an infinitesimally small initial disturbance can cause a large wing response. The transition movement is bounded ($\leq c$), so that $\Delta\alpha_t$ is bounded and the oscillation will have a limit cycle. When transition occurs and the wing has low structural damping, it is possible to visualize the wing amplitude building up progressively until the transition point actually reaches a point close to the leading edge on alternate surfaces during one complete cycle. This corresponds with the maximum wing response. The wing amplitude might then either reach a safe limit cycle or become sufficiently large to cause structural damage.[8,9]

As already noted, the change in incidence $\Delta\alpha_t$ is due primarily to changes in displacement surface camber. An increase in the severity of the adverse pressure gradient over the rear of a wing magnifies this effect as compared to a flat plate. This is illustrated in Fig. 9, which shows the large change in the boundary-layer displacement thickness calculated for RAE 5237 at $M = 0.8$ with transition at $x/c = 0.05$ and 0.40. Clearly, a quasisteady movement of the transition point over this distance will produce a large change in the displacement surface camber. Hence, wings with a strong tendency toward trailing-edge separation are likely to have larger values of $\Delta\alpha_t/\Delta\alpha$ and hence greater reduction in damping according to Eq. (2). This explains the difference in response between the sections shown in Fig. 5.

If the adverse pressure gradient over the wing is sufficiently strong to cause a laminar separation without a turbulent reattachment on the wing, then transition movements caused by the wing bending might be sufficient to suppress completely the separation on one surface during the cycle. This would cause a large change in the effective camber of the displacement surface and, hence, make a correspondingly large change in $\Delta\alpha_t/\Delta\alpha$ in Eq. (2). However, there is no evidence that this occurred within the present tests.

This discussion assumes that the large changes in response observed in the transition region are due primarily to reductions in damping. In addition to changes in the magnitude of the damping, an equally important effect would be changes in *phase angle,* but these cannot be quantified by quasisteady arguments.

According to quasisteady theory (as presented here), there is a direct equivalence between the flows associated with heaving and pitching motions. Hence, the flow sketches of Fig. 8 could give a physical explanation for losses in pitch damping observed with transitional boundary layers.[17-19]

It should be recalled that the bending motion of swept wings generates both heaving and pitching for any streamwise section.[20] According to the quasisteady theory presented here, with transitional boundary layers both components of this motion can generate negative aerodynamic damping.

Conclusions

Transitional boundary layers can cause a loss of positive aerodynamic damping or even the occurrence of negative aerodynamic damping, and a mechanism to explain this is proposed. The measurements currently available[4,5,7,9,17-19,21] suggest that the oscillations can occur over a wide range of frequency parameter and have important implications.

Force Tests in Ordinary and Cryogenic Wind Tunnels

Even on ordinary models in conventional wind tunnels, experience has shown that large-amplitude oscillations may occur with transitional boundary layers. These oscillations will be much smaller with large structural damping. (This is discussed in the Appendix.) For tests in cryogenic tunnels, the aerodynamic damping is likely to be more important than structural damping, even on ordinary models,[25] and oscillations must be expected with transitional boundary layers.

Buffeting Tests

The increase in response observed in the transition region could be important for buffeting measurements in high-speed wind tunnels. Suppose buffeting measurements were attempted on the steel wing of Fig. 7 with partially fixed transition at a Reynolds number as low as $R\bar{c} = 2 \times 10^6$. Then the buffet onset point could be masked by a high initial level of response. This masking would become serious if the initial response was used to obtain buffeting coefficients[22]. For the same model at lift coefficients higher than 0.40, transitional boundary layers could alter both the excitation and the

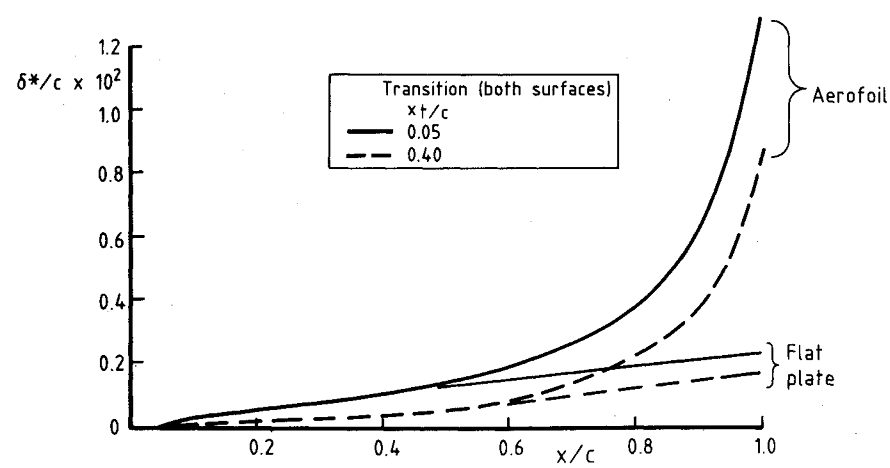


Fig. 9 Calculations of effect of transition position on distribution of displacement thickness along chord: aerofoil section RAE 5237, $M = 0.8$, $Re = 6.7 \times 10^6$, $C_L = 0$.

aerodynamic damping of the separated flows. Then extrapolation of buffeting measurements from $R\bar{c}=2\times10^6$ to full scale would be impossible. A possible procedure might be to make additional comparative buffeting measurements at, say, both $R\bar{c}=3\times10^6$ and 5×10^6 to check both the onset and severity of buffeting.

Flutter Tests

Flutter models are generally tested at constant Mach number over a wide range of equivalent airspeeds (EAS). In a conventional wind tunnel, this range of EAS inevitably corresponds with a wide range of Reynolds number. If the roughness is not sized for the lower Reynolds number, anomalous oscillations may occur.[8,21]

Laminar-Flow Aircraft

It is probable that laminar flow occurs over the complete span of some high-performance gliders. These have flexible wings of high aspect ratio with a small chord and often have low structural damping. It would be worth examining if high-performance gliders that have laminar flow have encountered any anomalous wing oscillations.

On larger aircraft, laminar flow can be maintained to higher Reynolds numbers by distributed suction. Under some conditions, such as pump failure or flying through turbulence, it may be possible for the boundary layer to become transitional, which might lead to wing oscillations. In flight tests in only light turbulence, no oscillations should be experienced with either fully turbulent or fully laminar boundary layers according to the evidence presented here. However, flight through heavy turbulence with laminar boundary layers might involve the development of transitional boundary layers if the initial wing amplitudes exceed certain levels. Designers of aircraft with significant areas of laminar flow should be aware of this problem.

Wing-Tip Vibration Absorbers

The large oscillations due to transitional boundary layers on a wind-tunnel model were attenuated by a simple form of vibration absorber. Such a device might be applicable to full-scale aircraft with wing-tip fuel tanks.[6]

Appendix: Wing Response to Unsteady Flow

Assume that there are no scale effects on the flow unsteadiness in the wind tunnel, as previous measurements have suggested.[14] Then, the forcing function in the first wing bending mode $\sqrt{nG(n)}$ will be constant at least outside the transition region and, provided $\xi\geq0$, the model response is given by[23,24]

$$\ddot{y}=(\sqrt{\pi}/2)\sqrt{nG(n)}\,[qS/A]\cdot\xi^{-1/2} \tag{A1}$$

The wing response $\ddot{y}$ may be measured directly by an accelerometer or more easily inferred from unsteady wing-root strain signals. Following the normal notation,[23] let the aerodynamic and structural damping coefficients be γ and $g/2$.

If structural damping is large compared with aerodynamic damping, as on steel models with many joints and if the model is tested at low pressures, then $\xi\simeq g/2$. The model response at constant Mach number in a conventional wind tunnel is then given by Eq. (A1) as

$$\ddot{y}\propto\rho/\sqrt{g} \quad \text{or} \quad \propto R/\sqrt{g} \tag{A2}$$

for a fixed model size, where Reynolds number is increased by increasing total pressure (or density) at constant total temperature and Mach number, $R\propto\rho$. In a cryogenic tunnel, it will often be convenient to increase Reynolds number at constant Mach number and kinetic pressure by holding the total pressure constant and reducing the total temperature.

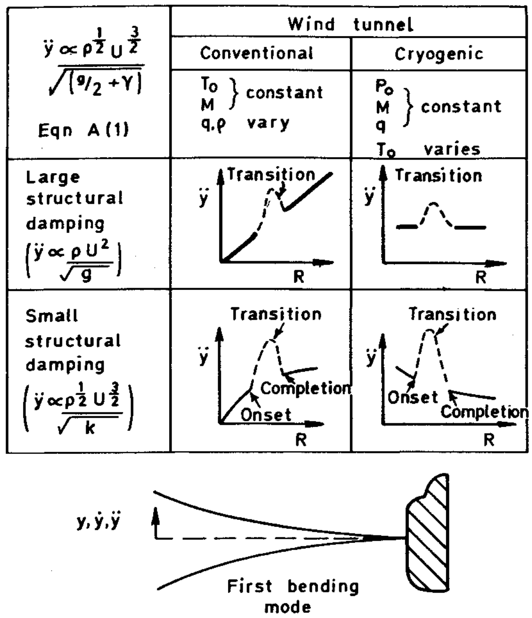


Fig. A1 Variation of response to flow unsteadiness with unit Reynolds number.

Then, according to Eq. (A1) and the assumption of constant and large structural damping, the model response will be independent of Reynolds number.

In contrast, if outside the region of transitional boundary layers structural damping is small compared with aerodynamic damping, as for "solid" aluminum or magnesium models,[12] then $\xi\simeq\gamma$. Where the aerodynamic damping is independent of the state of the boundary layer, it is proportional to the product of the freestream density and velocity, so that

$$\gamma=k\rho U \tag{A3}$$

where $k=\text{const}$ for a particular Mach number. Hence, from Eq. (A1), at constant Mach number in a conventional wind tunnel,

$$\ddot{y}\propto\rho^{1/2}U^{3/2}/\sqrt{k} \quad \text{or} \quad \propto\sqrt{R/k} \tag{A4}$$

In contrast, in a cryogenic wind tunnel operated at constant Mach number and total pressure, the model response according to Eq. (A1) is

$$\ddot{y}\propto T^{1/4}/\sqrt{k} \tag{A5}$$

Now, if the viscosity μ is $\propto T^{0.76}$ (a common approximation), the model Reynolds number is

$$R\propto T^{-1.26} \tag{A6}$$

at constant Mach number and total pressure, so that

$$\ddot{y}\propto(R)^{-0.20}/\sqrt{k} \tag{A7}$$

Figure A1 sketches the variation of wing response with Reynolds number for constant structural ($g/2$) and aerodynamic damping (k) coefficients according to these equations.

If in the region of transitional boundary layers there is a loss of aerodynamic damping, then k is reduced and there will be a marked increase in response, as indicated by the dotted curves. This increase should be particularly noticeable in a cryogenic tunnel, where the general level of response falls as Reynolds number increases [Eq. (A7)] roughly in proportion with the variation in turbulent skin friction (also about $R^{-0.20}$).

References

[1]Scruton, C. and Lambourne, N.C., "Similarity Requirements for Flutter Model Testing," *AGARD Manual of Aeroelasticity,* Pt. 4, Chap. 6, Sec. 2.3.

[2]Lambourne, N.C., "Flutter in One Degree of Freedom," *AGARD Manual of Aeroelasticity,* Pt. 5, Sec. 6.4.

[3]Ashill, P.R. and Fulker, J.L., "An Investigation of the Flow Around a Swept Wing Section with Differing Rear Pressure Rises," RAE Rept., to be published.

[4]Ashill, P.R. and Weeks, D.J., "An Experimental Investigation of the Drag of Thick Supercritical Aerofoils—a Progress Report," RAE Tech. Mem., Aero 176, 1978.

[5]Mabey, D.G. and Ashill, P.R., "On Aeroelastic Oscillations Associated with Transitional Boundary Layers," RAE Tech. Memo., Aero 1995, Feb. 1984.

[6]Mabey, D.G., "Aircraft 1° Freedom Flutter Damper," British Patent Application 84-18816, July 24, 1984.

[7]Mabey, D.G., Welsh, B.L., and Cripps, B.E., "Further Aerodynamic Characteristics of Moving Trailing-Edge Controls at Subsonic and Transonic Speeds," RAE Tech. Rept. 80134, 1980.

[8]Houwink, R., Kraan, A.N., and Zwaan, R.J., "Wind-Tunnel Study of the Flutter Characteristics of a Supercritical Wing," *Journal of Aircraft,* Vol. 19, May, 1984, pp. 400–405.

[9]Erickson, L.L., "Transonic Single-Mode Flutter and Buffet of a Low Aspect Ratio Wing Having a Subsonic Aerofoil Shape," NASA TND 7346, 1974.

[10]Martin, L., BAE Unpublished Tests.

[11]Ericsson, L.E., "Effect of Boundary Layer Transition on Vehicle Dynamics," *Journal of Spacecraft and Rockets,* Vol. 6, Dec. 1969, pp. 1404–1409.

[12]Mabey, D.G. and Butler, G.F., "Measurements of Buffeting on Two 65° Delta Wings of Different Materials," AGARD CP 266, 1977, Paper 6.

[13]Den Hartog, J.P., *Mechanical Vibration,* McGraw-Hill, New York, 1956.

[14]Mabey, D.G., "Flow Unsteadiness and Model Vibration in Wind Tunnels at Subsonic and Transonic Speeds," British Aeronautical Research Council, CP 1155, 1971.

[15]Preston, J.H., "Calculation of Lift Taking Account of the Boundary Layer," British Aeronautical Research Council, R&M 2725, 1949.

[16]Spence, D.A. and Beasley, J.A., "Calculation of Lift Slopes Allowing for Boundary Layers with Applications to RAE 101 and 104 Aerofoils," British Aeronautical Research Council, R&M 3137, 1958.

[17]Greidanus, J.H., Van de Vooren, A.I., and Bergh, H., "Experimental Determination of the Aerodynamic Coefficients of an Oscillating Wing in Incompressible Two-Dimensional Flow, Part I: Wing with Fixed Axis of Rotation," NLL Rept. F101, 1952.

[18]Van de Vooren, A.I. and Bergh, H., "Spontaneous Oscillations of an Aerofoil due to Instability of the Laminar Boundary Layer," NLL Rept. F96, 1951.

[19]Lambourne, N.C., "An Experimental Investigation of the Flutter Characteristics of a Model Flying Wing," British Aeronautical Research Council, R&M 2626, 1947.

[20]Fung, Y.C., *An Introduction to the Theory of Aeroelasticity,* Wiley, New York, 1955, Eq. (5), Sec. 4.4.

[21]Persoon, A.J., Horsten, J.J., and Meijer, J.J., "On Measuring Transonic Dips in the Flutter Boundaries of a Supercritical Wing in the Wind Tunnel," NLR MP 83-088 U, May 1983.

[22]Mabey, D.G., "An Hypothesis for the Prediction of Flight Penetration of Wing Buffeting from Dynamic Tests on Wind Tunnel Models," British Aeronautical Research Council, CP 1171, 1971.

[23]Davis, D.D. and Wornom, D.E., "Buffet Tests on an Attack Airplane Model with Emphasis on Data from Wind Tunnel Tests," NACA RM L57 H13, 1958 (NASA TIL 6772).

[24]Jones, J.G., "A Survey of the Dynamic Analysis of Buffeting and Related Phenomena," RAE Tech. Rept. 72197, 1973.

[25]Mabey, D.G., "Some Remarks on Dynamic Aeroelastic Model Tests in Cryogenic Wind Tunnels," NASA CR 145029, 1975.